CBA Algebra I

Standard

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Numbers:

3.1415926535897932
3846264338327950288
4197169399375105820
9749445923078164
0628620899862803482
534211706798214808
6513282306647093844
6095505822317253594
0812848111745028410
2701938521105559644
6229489549303819644
2881097566593344612
8475648233786783165
2712019091456485669
234603486

Care for some pi?

Questions:

1) **How many numbers are there?**
   1b) **Why?**

2) **How many types or classes of numbers are there?**

3) **How could we count sheep and other *N*atural things?**

4) **How can negative ☹️ integers (*Z* for *Z*ahlen[1]) help us understand hitting rock bottom?**

5) **Going for a pizza, what's a rational slice for each friend?**

6) **What's a transcendental way for a ratio to be *R*eal?** [2]

7) **What's on the Horizon of Complete Complexity?** [3]

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[1] Zahlen is "number" in German. This is why: *Z* , represents the integers in English.
[2] A complete answer is beyond the scope of this class and will be found within an upper level undergraduate course.
[3] same as footnote 2

Our number systems in fancy math symbols:

$$N \subset Z \subset Q \subset R \subset C$$

Naturally this looks odd at first. Imagine Russian egg dolls. N is inside of Z, which is inside of Q, which is inside of R, which is inside of the large C.

The set C is the set of all Complex numbers. They feature: *i* . *i* = $\sqrt{(-1)}$. That's the square root of negative one. You will see *i* in the Schrödinger equation in university.

R is the set of all Real numbers. It is essentially all the numbers you know and will need in life, unless you become someone like a physicist -only then will you need to know about C.

Q is the set of all Rational numbers. It sounds unusual at first, but they're basically integers *and* conventional looking fractions.

Z is the integers, N is the natural numbers.

Variables & Labels:

Can we compare apples to oranges?

Questions:

1) **What is a variable and what does it represent?**
   b) How can its value vary?

2) **Can we add or multiply apples with oranges?**

Imagine a bag of groceries has: one sandwich, one juice, and two pieces of fruit. How many pieces of fruit are there in total if we possess 3 bags.

3 bags = 3 * (S + J + 2*F) = 3*S + 3*J + 6*F .

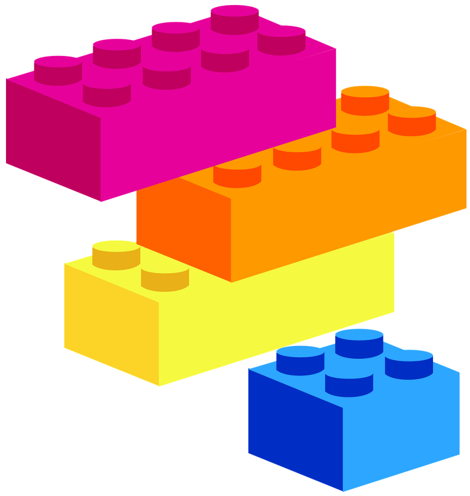
So 6 pieces of fruit.

Consider the Grocery function. It is essentially a machine that takes in inputs: number of bags considered. Ratios are preserved. The function then *maps* out the number of total items.

G(x) = 4*x + 0 .

Zero is our y intercept. This equation graphs a line that hits the y and x axes at the *origin*.

Prime Factorization:



Consider the *Fundamental Theorem of Arithmetic*:

Every integer *n* > 1, can be written with a unique signature of *primes* multiplied together. A prime is just a number that is divisible by itself and 1.

Examples: (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 42, 47, …) .

$$n = p_1^{e1} * \ldots * p_k^{ek} .$$

In the equation *e* is an exponent and it takes on an integer potentially equal to 0 or more. *p* represents a potential prime number.

For Example:

$$27 = 3^3 .$$

$$24 = 2^3 * 3 .$$

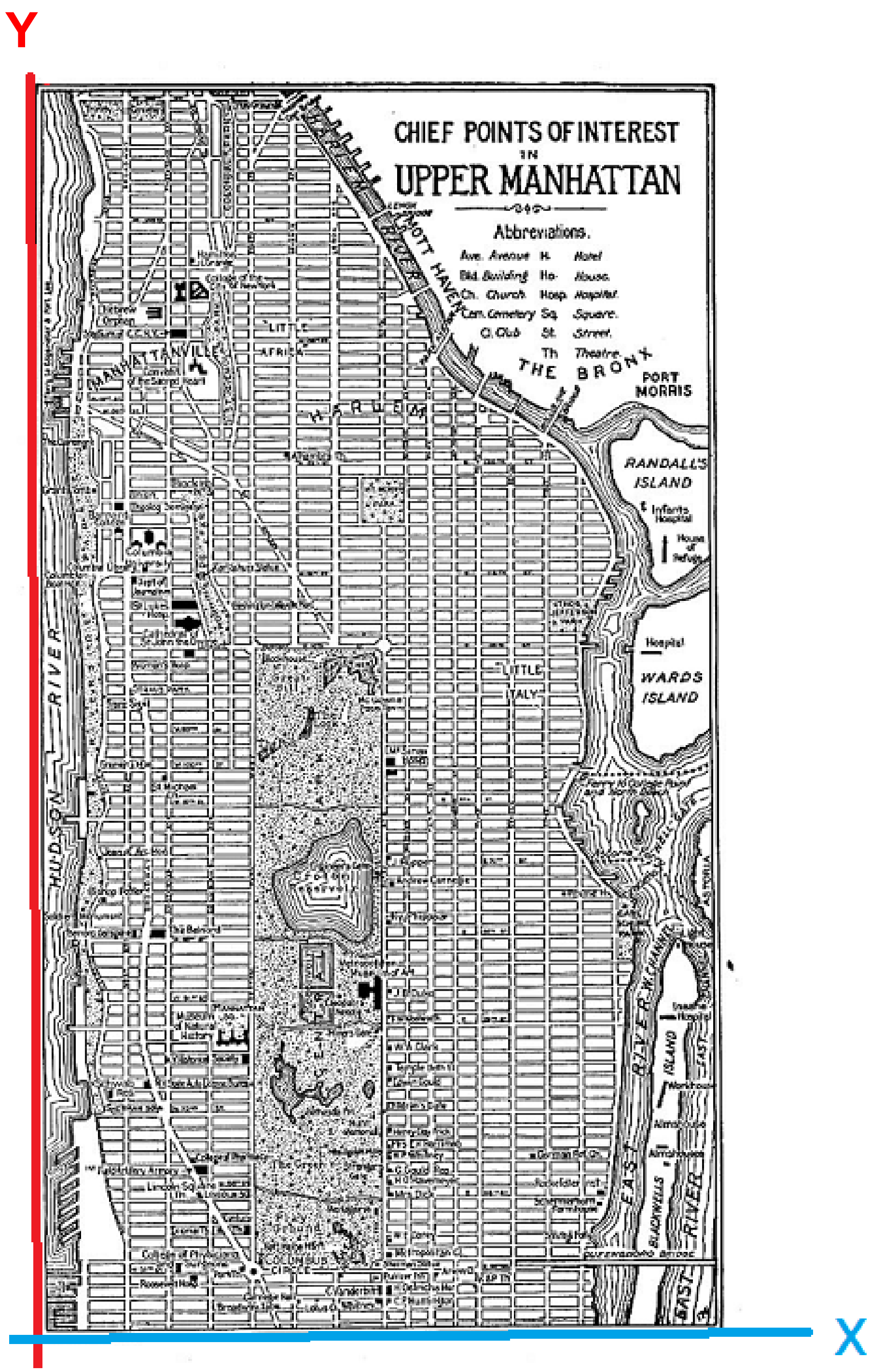
$$2 = 2^1 .$$

$$420 = 2^2 * 3 * 5 * 7 .$$

Can you factor: 625, 81, 37 , (160 / 5) ?

We can think of the primes as the building blocks of the *composite* (not prime) numbers.

Graphs:

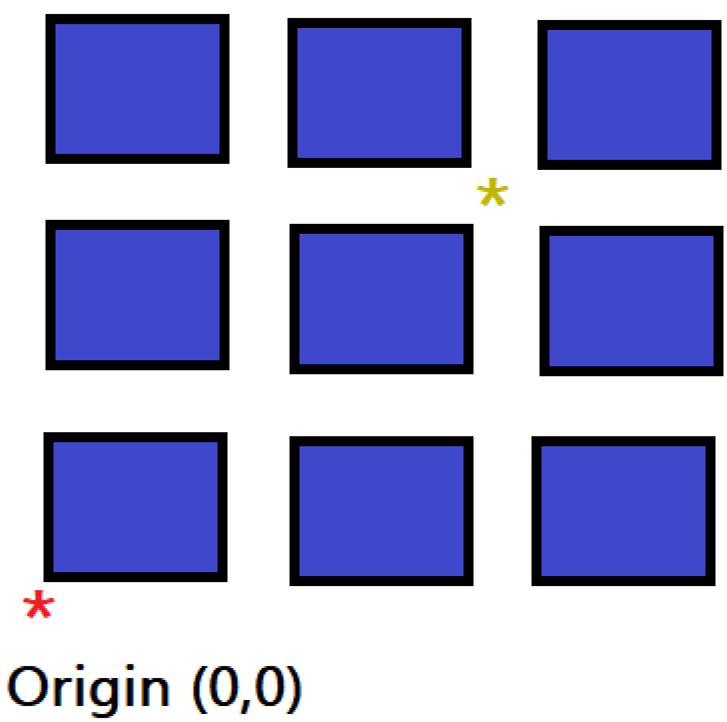


A city with DesCartes in mind…

So for those unfamiliar with the island of Manhattan, the streets and avenues run more less in accordance to an X & Y grid.

The Streets run parallel to the X axis. The avenues run parallel to the Y axis.

Questions:



Consider the above figure.

How many avenues to the right and streets up does one have to walk from the red star (where we originate) to our destination at the gold star?

(Remember the avenues run up and down and the streets run left to right.)

Graphing equations should be straight forward. You'll have more practice later. Make sure you practice graphing extra so you can breeze through those types of questions and focus on quadratic equations and the quadratic formula -when taking an exam. Those latter problems tend to be more challenging for students, both in terms of computation and conception.

Polynomials:

For our purposes, we will stay in 2 dimensions.

Consider the *Fundamental Theorem of Algebra:*

A polynomial of degree d, with real or complex *coefficients*, has <u>at most</u> d distinct *roots* (some in *R*, and all in *C* since *R* ⊂ *C*.)

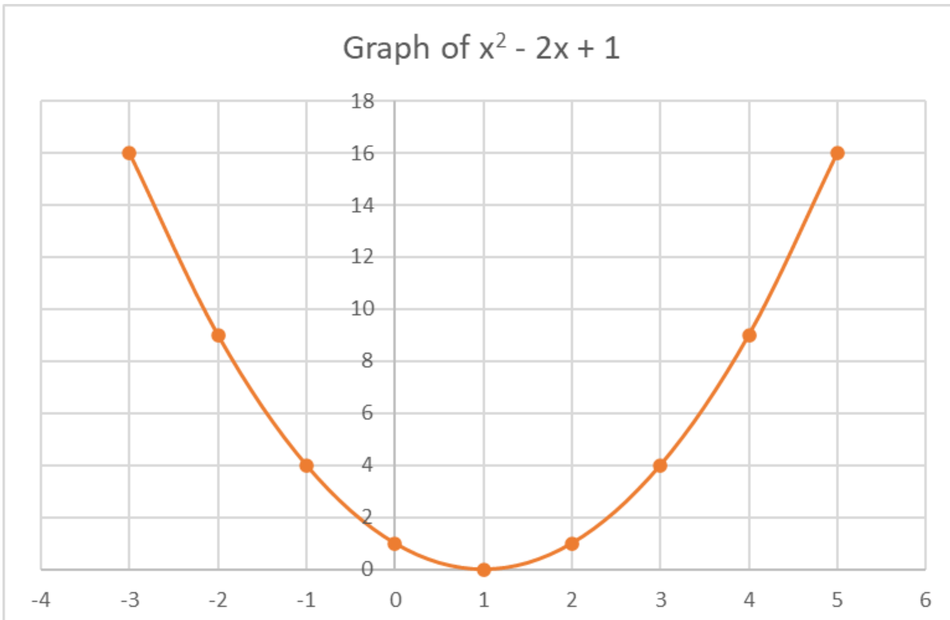
For example: $P(x) = x^2 - 2x + 1$ has 2 roots x = 1 and x = 1.

***Proof:* Let $x^2 - 2x + 1 = 0$,**

so then (x - 1)*(x - 1) = 0.

So x = 1 is our root, and we have two copies basically.

Now the above polynomial can be graphed as follows:



Notice our root (the point where the curve hits the x-axis) is at (1, 0). Notice the zero in the y coordinate slot. That's why we set the polynomial equal to zero and solve for our x coordinates. The x coordinates that we find are called the roots of our polynomial.